The study of the car's stability using a simplified model

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Abstract. This paper presents some research on vehicle stability, taking into account a simplified mechanical model of cars and analyzing the stability of vehicle movement using dynamic systems theory. The paper proposes a new mathematical model for the kinematic and dynamic study of the behavior of two-axle vehicles, a model using a nonlinear link between the lateral deflection of the tire and the external forces applied to it. The mechanical model is considered a model with two degrees of freedom, taking into account the following two variables: angular velocity and linear velocity in the transverse direction. To analyze the dynamic behavior of the vehicle, two displacement situations are considered: the first case in a case of displacement with a constant turning angle and the second considered case is the case with the sinusoidal variation of the wheel turning angle. Numerical solutions for these driving situations are determined and appreciations are made regarding the character of the movement in terms of stability.

1. Introduction

The automotive moving and its stability are significant problems for automotive constructors, whom permanent watching the customers interests. Those requirements are touching dynamic performances, design, mobile security, amenities. Various mechanical models for the study of dynamic behavior are proposed in the literature [1], [6], [7], [8], mechanical models with one to six degrees of freedom.

With this paper we purpose to realize a study over automotive stability using the dynamic systems theory [9], under a simplified form of plan automotive model.

2. The simplified mechanical and mathematical model for vehicles plane moving the study

The mathematical models used in the vehicle dynamics study contain a number of smaller or larger variables [2], [3], [4] in accordance with the relative magnitude in the dynamics analyzed. In this study the next degree of freedom is taking into account:

- x, define the displacement along longitudinal axle;
- y, define the displacement along transversal axle;
- φ, define the circular displacement of suspended mass around Oz axle;

Towards to obtain a simplified mechanical and mathematical model, who can be resolved both using a computer and analytical method, some simplified hypothesis may be considerate:

- The vertical forces reactions abide constant, this means that no redistribution of those are occurring by reason of rolling and pitching oscillations;
- The rolling and pitching displacement are neglected;
- No longitudinal forces are acting on tires (the free tire case);

2.1. Simplified plan model

A simplified plan model of the vehicle is presented in Figure 1, and the next notations are used:
a, b – are the coordinates of mass center point towards front axle, respectively rear axle;
F1, F2 – is the longitudinal forces of the contact area between tire and road;
Y1, Y2 – are the lateral forces of the contact area between tire and road;
δ_1 - is the cornering angle;
α_1, α_2 - are the sleep angles for the front axle, respectively rear axle (Figure 1).

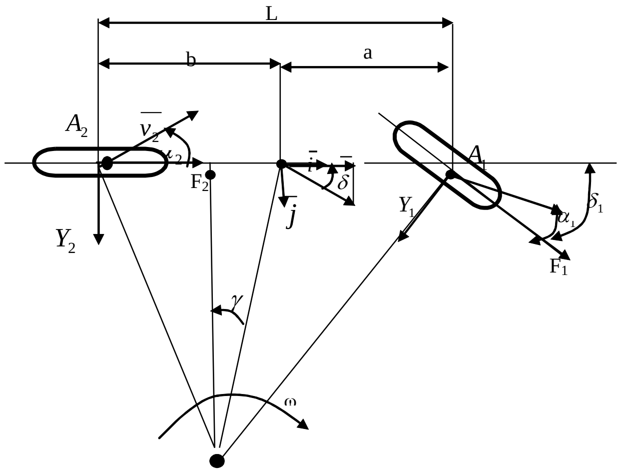


Figure1. The simplified model of vehicle

2.2. The mathematical model

The velocity of points A1, A2 are:

$$\vec{v}_{A_1} = \vec{v}_C + \vec{\omega} \times \overrightarrow{CA_1} = v_x \cdot \vec{i} + v_y \cdot \vec{j} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & \omega \\ a & 0 & 0 \end{vmatrix} \tag{1}$$

where ω is the yaw angular speed

v_x – is the longitudinal velocity of the vehicle
v_y – is the transverse velocity of the vehicle

$$\vec{v}_{A_2} = \vec{v}_C + \vec{\omega} \times \overrightarrow{CA_2} = v_x \cdot \vec{i} + v_y \cdot \vec{j} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & \omega \\ -b & 0 & 0 \end{vmatrix} \tag{2}$$

$$v_{1x} = v_{2x} = v \tag{3}$$

$$v_{1y} = v_y + a\omega \tag{4}$$

$$v_{2y} = v_y - b\omega \tag{5}$$

$$tg(\delta_1 - \alpha_1) = \frac{v_{1y}}{v_{1x}} = \frac{v_y + a\omega}{v} \quad (6')$$

$$\alpha_1 = \delta_1 - arctg\frac{v_y + a\omega}{v} \quad (6)$$

$$tg\alpha_2 = -\frac{v_{2y}}{v_{2x}} = -\frac{v_y - b\omega}{v} \quad (7')$$

$$\alpha_2 = -arctg(\frac{v_y}{v} - \frac{b\omega}{v}) \quad (7)$$

$$\vec{a} = \frac{d\vec{v}}{dt} + \vec{\omega} \times \vec{v}\ ; \quad a_x = \frac{dv_x}{dt} - v_y \cdot \omega\ ; \quad a_y = \frac{dv_y}{dt} + v \cdot \omega \quad (8)$$

The equations for car's moving are:

$$\begin{cases} m\left(\dot{v}_y + v\omega\right) = Y_1 \cos\delta_1 + Y_2 + F_1 \sin(\delta_1) \\ I_z \cdot \dot{\omega} = Y_1 \cos\delta_1 \cdot a - Y_2 \cdot b + F_1 \sin(\delta_1) \end{cases} \quad (9)$$

The lateral forces are given by the next relation:

$Y_{1,2} = k_{1,2} \cdot \alpha_{1,2}$, where $k_{1,2}$ – lateral stiffness coefficient of the tire;

$$\begin{cases} m\left(\dot{v}_y + v\omega\right) = k_1\left(\delta_1 - arctg\left(\frac{v_y + a\omega}{v}\right)\right)\cos\delta_1 + k_2 \cdot arctg\left(\frac{v_y - b\omega}{v}\right) + F_1 \sin(\delta_1) \\ I_z \cdot \dot{\omega} = k_1\left(\delta_1 - arctg\left(\frac{v_y + a\omega}{v}\right)\right)\cos\delta_1 \cdot a - k_2 \cdot arctg\left(\frac{v_y - b\omega}{v}\right) \cdot b + F_1 \sin(\delta_1) \cdot a \end{cases} \quad (10)$$

For small value of angular lateral deformation we can make the next approximation:

$$\begin{gathered} arctg\frac{v_y + a\omega}{v} \cong \frac{v_y}{v} + \frac{a}{v} \cdot \omega \\ arctg\frac{v_y - b\omega}{v} \cong \frac{v_y}{v} - \frac{b}{v} \cdot \omega \\ \cos\delta_1 \approx 1 \end{gathered} \quad (11)$$

The next system is obtained:

$$\begin{cases} \dot{v}_y = -\dfrac{k_1 + k_2}{mv} \cdot v_y - \left(\dfrac{k_1 a - k_2 b}{mv^2} + 1 \right) \cdot \omega + \dfrac{k_1}{mv} \cdot \delta_1 \\ \dot{\omega} = -\dfrac{k_1 a - k_2 b}{I_z \cdot v} \cdot v_y - \dfrac{k_1 a^2 + k_2 b^2}{I_z \cdot v} \cdot \omega + \dfrac{k_1 a}{I_z} \cdot \delta_1 \end{cases} \tag{12}$$

In the case of constant velocity, a non-homogeneous system with constant coefficients is obtained.

Using the matrices form, the system may be written:

$$\{\dot{x}\} + [A]\{x\} = \{B\}, \tag{13}$$

with: $\{x\} = \{v_y, \omega\}^T$ - is the velocity vector

$\{\dot{x}\} = \left\{ \dot{v}_y, \dot{\omega} \right\}^T$ - is the acceleration vector

$$[A] = \begin{bmatrix} \dfrac{k_1 + k_2}{mv} & \dfrac{k_1 a - k_2 b}{mv^2} + 1 \\ \dfrac{k_1 a - k_2 b}{I_z} & \dfrac{k_1 a^2 + k_2 b^2}{I_z v} \end{bmatrix} \qquad \{B\} = \begin{bmatrix} \dfrac{k_1}{mv} \cdot \delta_1 \\ \dfrac{k_1 a}{I_z} \cdot \delta_1 \end{bmatrix} \tag{14}$$

If the variation of the cornering angle is known, the analytical solution may be founded. The next case may be taken into account:

- $\delta_1(t) = \delta_{10} = const$ (this case corresponds with cornering with constant radius)

$$\begin{pmatrix} \delta(t) \\ \omega(t) \end{pmatrix} = \begin{pmatrix} \dfrac{k_1}{mv} \delta_{10} \cdot t \\ \dfrac{k_1 a}{I_z} \delta_{10} \cdot t \end{pmatrix} - \begin{bmatrix} \dfrac{k_1 + k_2}{mv} & \dfrac{k_1 a - k_2 b}{mv^2} + 1 \\ \dfrac{k_1 a - k_2 b}{I_z} & \dfrac{k_1 a^2 + k_2 b^2}{I_z v} \end{bmatrix} \cdot \begin{pmatrix} \dfrac{t^2}{2} \cdot \dfrac{k_1}{mv} \delta_{10} \\ \dfrac{t^2}{2} \cdot \dfrac{k_1 a}{I_z} \delta_{10} \end{pmatrix}$$
$$+ \frac{1}{2} \cdot \begin{bmatrix} \dfrac{k_1 + k_2}{mv} & \dfrac{k_1 a - k_2 b}{mv^2} + 1 \\ \dfrac{k_1 a - k_2 b}{I_z} & \dfrac{k_1 a^2 + k_2 b^2}{I_z v} \end{bmatrix}^2 \cdot \begin{pmatrix} \dfrac{t^3}{3} \cdot \dfrac{k_1}{mv} \delta_{10} \\ \dfrac{t^3}{3} \cdot \dfrac{k_1 a}{I_z} \delta_{10} \end{pmatrix} + \ldots \tag{15}$$

The homogeneous system solution has the form:

$$\{x\} = e^{-[A]t} \cdot \{c\} \tag{16}$$

$\{c\} = \{c_1 \quad c_2\}^T -$ which depend on the initial conditions,

For the non-homogeneous system the solution is:

$$\{x\} = e^{-[A]t} \cdot \{c\} + \int_0^t e^{-[A](t-\tau)} \cdot B(\tau) d\tau \tag{17}$$

$$\begin{pmatrix}\delta\\ \omega\end{pmatrix} = e^{-A[t]}\cdot\begin{pmatrix}c_1\\ c_2\end{pmatrix} + \int_0^t e^{-[A](t-\tau)}\cdot\begin{pmatrix}\frac{k_1}{mv}\cdot\delta_1(\tau)\\ \frac{k_1 a}{I_z}\cdot\delta_1(\tau)\end{pmatrix}d\tau \tag{18}$$

For the initial conditions $\delta = 0$ and $\omega = 0$, $c_1 = 0$ and $c_2 = 0$ are obtained and using the serial extension of e^{At} function, the system solution is obtained:

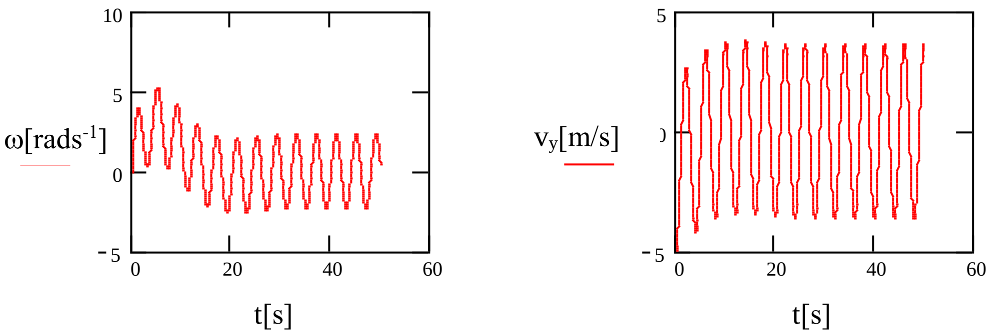
$$\begin{pmatrix}\delta\\ \omega\end{pmatrix} = \int_0^t \left\{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} - \frac{1}{1!}A(t-\tau) + \frac{1}{2!}A^2(t-\tau)^2 - \frac{1}{3!}A^3(t-\tau)^3 + \ldots\right\}\cdot\begin{pmatrix}\frac{k_1}{mv}\delta_1(\tau)\\ \frac{k_1 a}{I_z}\delta_1(\tau)\end{pmatrix}d\tau \tag{19}$$

The analytical solution is obtained by knowing the law of variation of the angle of rotation. It is considered the variation of the steering angle has $\delta_1(t) = A\sin(\omega\cdot t)$ form and the next solution is obtained:

$$\begin{aligned}\begin{pmatrix}\delta(t)\\ \omega(t)\end{pmatrix} = &\begin{pmatrix}\frac{k_1\cdot A}{mv\omega}\cdot(-\cos(t\cdot\omega)+1)\\ \frac{k_1 a\cdot A}{I_z\cdot\omega}\cdot(-\cos(t\cdot\omega)+1)\end{pmatrix} - \begin{bmatrix}\frac{k_1+k_2}{mv} & \frac{k_1 a - k_2 b}{mv^2}+1\\ \frac{k_1 a - k_2 b}{I_z} & \frac{k_1 a^2 + k_2 b^2}{I_z v}\end{bmatrix}\cdot\begin{pmatrix}\frac{k_1}{mv}\cdot\frac{A}{\omega}\cdot\left(\frac{-\sin(\omega t)}{\omega}+t\right)\\ \frac{k_1 a}{I_z}\cdot\frac{A}{\omega}\cdot\left(\frac{-\sin(\omega t)}{\omega}+t\right)\end{pmatrix}\\ &+\frac{1}{2}\cdot\begin{bmatrix}\frac{k_1+k_2}{mv} & \frac{k_1 a - k_2 b}{mv^2}+1\\ \frac{k_1 a - k_2 b}{I_z} & \frac{k_1 a^2 + k_2 b^2}{I_z v}\end{bmatrix}^2\cdot\begin{pmatrix}\frac{k_1}{mv}\cdot\frac{A}{\omega^3}\cdot\left(2\cos(\omega t)+\omega^2 t^2-2\right)\\ \frac{k_1 a}{I_z}\cdot\frac{A}{\omega^3}\cdot\left(2\cos(\omega t)+\omega^2 t^2-2\right)\end{pmatrix}+\ldots\end{aligned} \tag{20}$$

In Figure 2 are presented the numerical solutions in the case of initial velocity v=25 km/h and the sinusoidal trajectory with a variation of the angle having the next form $\theta(t) = 15\cdot\sin(\frac{\pi}{2}\cdot t)$.

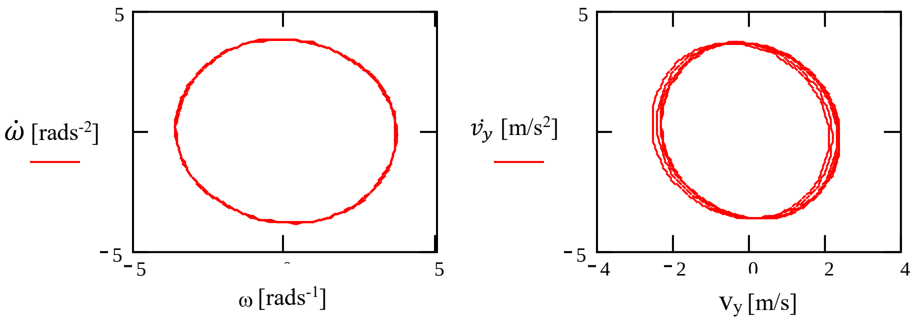
The numerical solutions are presented for the transverse velocity in Figure 2b and for angular velocity in Figure 2a.



a) **b)**

Figure 2 Numerical results in case of sinusoidal trajectory

The space of the phases is represented in the figure and shows the stable character of the motion of the analyzed system



a) b)

Figure 3. The space of phases in case of sinusoidal trajectory

3. CONCLUSION

This mathematical model is a simple but useful model for vehicle stability analysis, based on the theory of dynamic systems or the theory of classical mechanics. The values obtained are accurate values, compared to the measurements made on a real vehicle [2], having the same parameters as those used in the numerical simulation, so this can be considered a validation for this model.

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